Optimal & fast Wiener filtering of CMB maps without preconditioning

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Background

Wiener Filter (WF)

$$(\mathbf{S}^{-1} + \mathbf{N}^{-1})\mathbf{s}_{\mathrm{wf}} = \mathbf{N}^{-1}\boldsymbol{d}$$

- Signal reconstruction from noisy data
- **Optimal** data analysis solution for Gaussian Random Fields ≈ Cosmic density field
- Widespread **applications** in cosmology & astrophysics

WF equation → Numerical issues

$\mathbf{s}_{\mathrm{wf}}$: WF solution
S & N: Signal & Noise covariance
d: data set

- Inversion of dense matrices
- Not computationally tractable for modern data sets
- Standard method: Preconditioned Conjugate Gradient (**PCG**)
- But preconditioner required:
  Problem-dependent
  Ill-conditioning issue
  Computationally unstable
- Complex preconditioners: Multi-scale, multi-grid etc.

New Method

S & **N** sparse (diagonal) in **different bases**

→ **Introduce a messenger field to act as an intermediate between the 2 bases**

$\mathbf{T} = \min(\mathrm{diag}(\mathbf{N}))\mathbb{1}$, $\mathbf{U} = \min(\mathrm{diag}(\mathbf{S}))\mathbb{1}$, $\bar{\mathbf{S}} = \mathbf{S} - \mathbf{U}$, $\bar{\mathbf{N}} = \mathbf{N} - \mathbf{T}$, $\xi = \mathbf{T} + \mathbf{U}$

Pixel space $\left[\bar{\mathbf{N}}^{-1} + \xi^{-1}\right] \boldsymbol{t} = \bar{\mathbf{N}}^{-1}\boldsymbol{d} + \xi^{-1}\boldsymbol{s}$

Fourier space $\left[\xi^{-1} + \bar{\mathbf{S}}^{-1}\right] \boldsymbol{s} = \xi^{-1}\boldsymbol{t}$

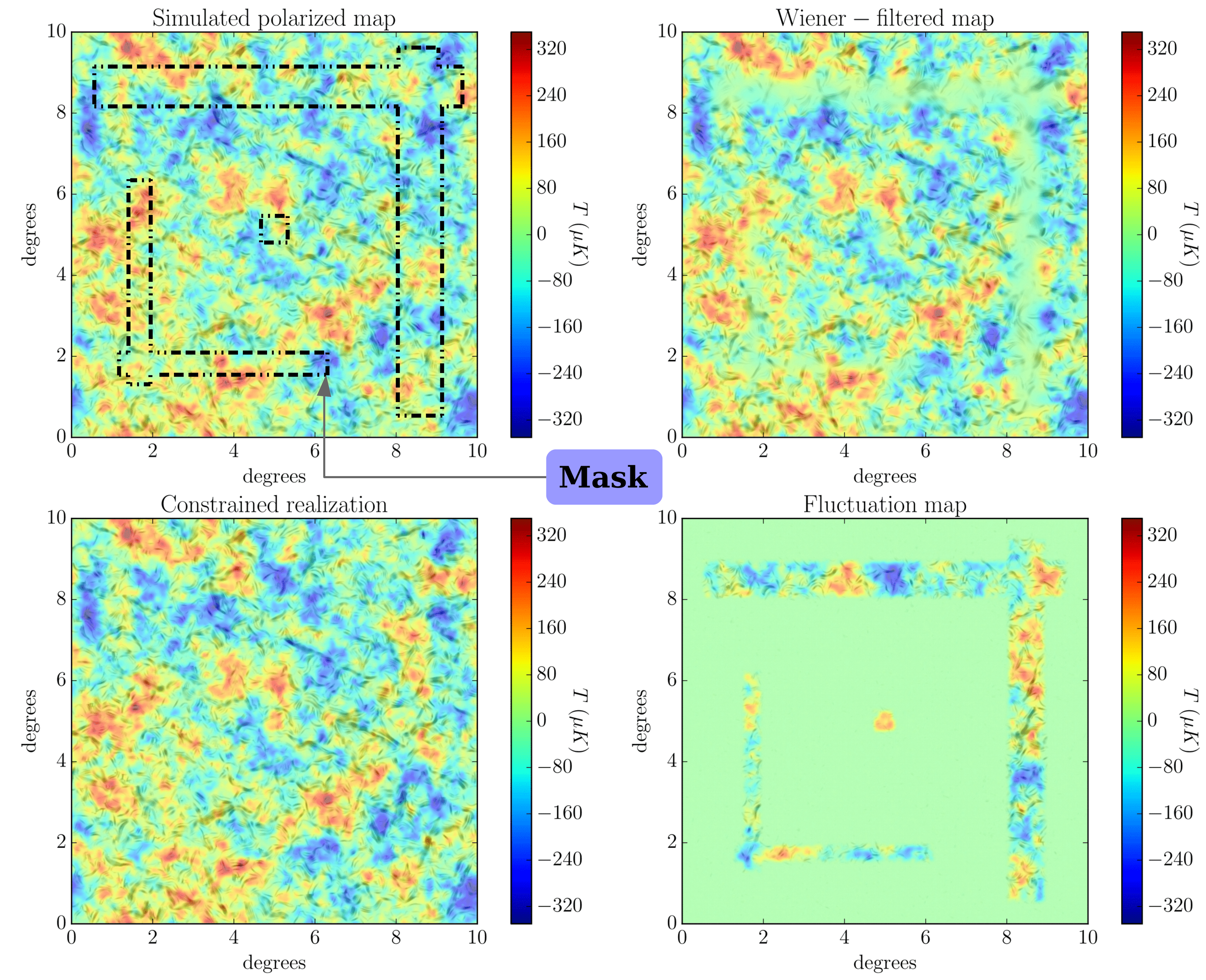
Solve 2 algebraic equations iteratively

Dual Messenger (DM) Algorithm

- **Preconditioner-free**
- **Hierarchical framework**

DKR, Lavaux & Wandelt (2017a, MNRAS)

Fig. 1: **Wiener-filtered maps & constrained realizations**



Simulation

DKR, Lavaux & Wandelt (2017b, in prep.)

1) Generate a polarized CMB map + contaminate with **correlated** noise
2) Apply mask as shown by dashed lines in Fig. 1
3) Compute **Wiener-filtered (WF)** map & **constrained realizations (CR)** using DM algorithm

- **Convergence diagnostics: High-quality DM solution**
- **Failure of PCG to converge to a plausible solution**
- **Execution time of few minutes on single core of standard workstation** ($\epsilon = 10^{-4}, N_{\mathrm{pix}} = 1024^2$)

- DM algorithm correctly reconstructs **unbiased** power spectra **(CR)** consistent with the input power spectra (Fig. 2).
- Yields a **full-sky noiseless** sample with trivial (leakage-free) *E*-*B* separation (Fig. 3).

Fig.2: **Power spectrum reconstruction**

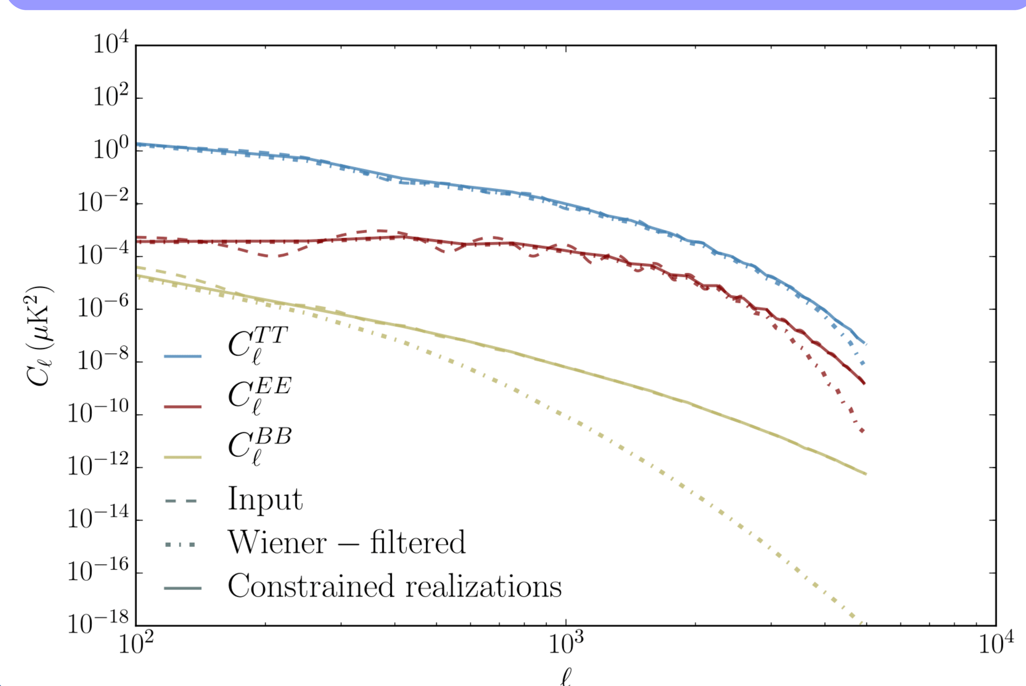
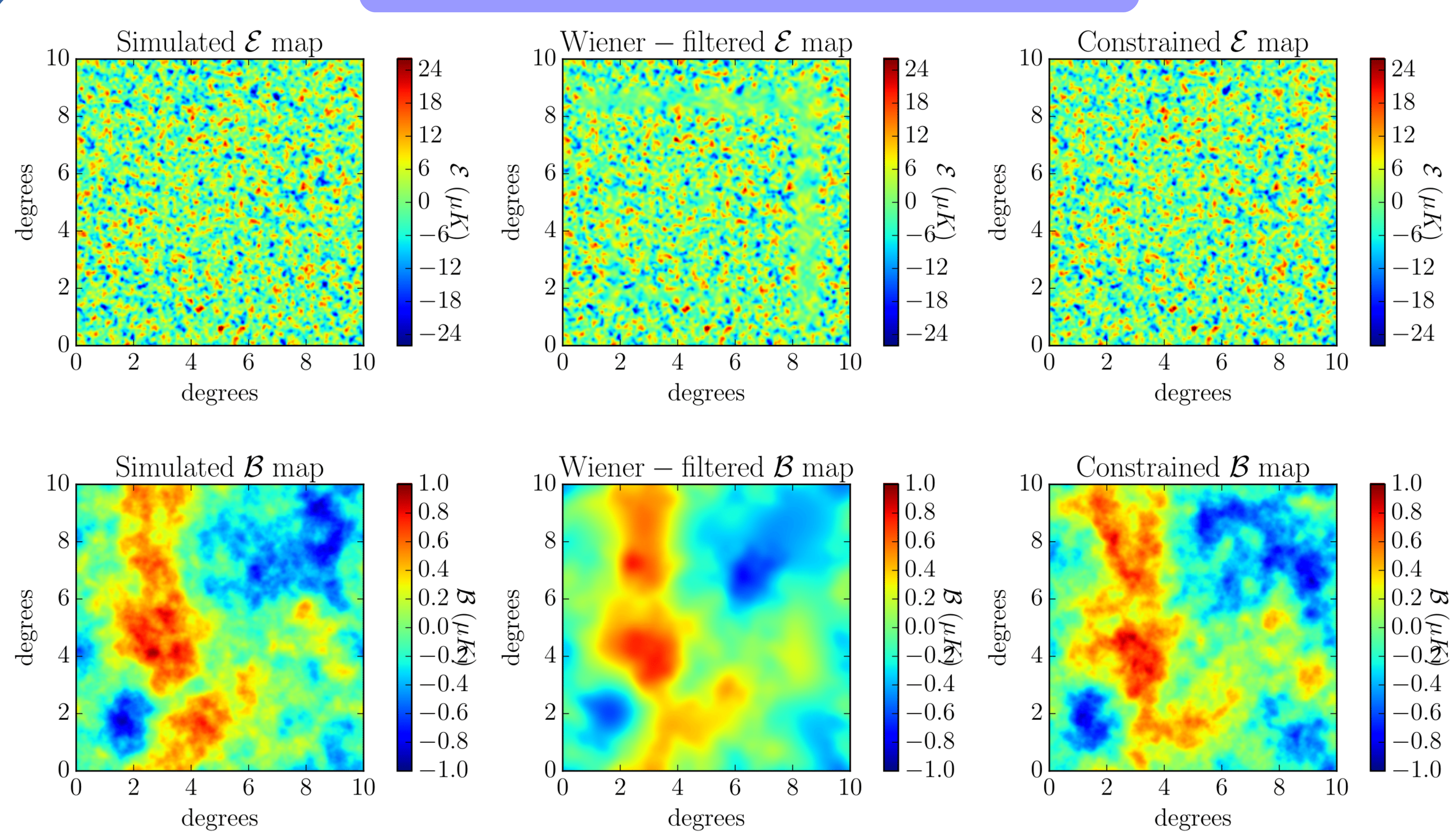


Fig.3: **Separation of *E*- & *B*-modes**



Conclusions

- **Fast & efficient** preconditioner-free Wiener filtering algorithm
- **Unconditionally stable**; guaranteed to converge
- **Straightforward/trivial** numerical implementation
- Deals **effectively** with ill-conditioned systems, essential for CMB polarization
- Adapted to generate **constrained realizations** consistent with observations
- Relevant for **current & next-generation** CMB experiments: Planck, CMB-S4, ...

- Renders **exact global Bayesian analyses** of high -resolution & high-sensitivity CMB observations numerically tractable and efficient (**Gibbs sampling**).
- **Statistically optimal separation** of pure *E*- & *B*-modes following prescription by Bunn & Wandelt (2016).
- DM algorithm can be augmented to deal with highly **complex noise models**.

References

- DKR, Lavaux & Wandelt, 2017a, MNRAS, 468, 1782 (arXiv:1702.08852)
- DKR, Lavaux & Wandelt, 2017b, in prep. (Stay tuned)
- Elsner & Wandelt 2013, A&A, 549, A111 (arXiv:1210.4931)
- Bunn & Wandelt 2016 (arXiv:1610.03345)

Background credit: hdimagelib.com