THE ROCHE COORDINATES IN NON-SYNCHRONOUS BINARIES

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Abstract. The Cayley–Darboux problem for the Roche model of binaries is reinvestigated. Generalised Roche coordinates are then defined and calculated in the form of power series of potential for the general case of non-synchronous binaries with eccentric orbits.

1. Introduction

The computation of models of stellar atmospheres and comparison of the corresponding synthetic spectra with observations is an important clue to the investigation of stars. The great progress in refining the microphysics in this field (e.g., opacities and statistical equilibrium) contrasts with the roughness of the methods used to incorporate the geometry and dynamics of atmospheres. The line profiles of rotating stars are usually calculated by convolution of rotational profiles with those radiated by the static star, despite the oblateness of the rotating star and variability of $T_{\rm eff}$ and g across its surface. These effects are occassionally included in modelling the tidally distorted components of binaries via the integration of spectra of plane-parallel models with $T_{\rm eff}$ and g related by the gravity-darkening law. However, due to von Zeipel's paradox, hydrostatic and radiative equilibria (commonly used in these models) cannot be simultaneously valid in distorted stars. Moreover, the geometrical thickness of the atmosphere need not be negligible in comparison with the curvature radius of the star's surface. Regarding the general importance of contact and semi-detached, as well as rapidly rotating stars, it would be desirable to improve the computing of their atmospheres both from the hydrodynamical point of view (e.g., calculation of meridional circulations and stellar wind) and in treating the radiative transfer under conditions of general (non-symmetrical) geometry and velocity field. To simplify this (generally 3-dimensional) problem, it is useful to treat it in coordinates fitting its approximate symmetry along the surface.

Under the condition of hydrostatic equilibrium, which can be accepted as a good zero-order approximation, the atmosphere must be homogeneous on each equipotential surface $\phi = \text{const}$. It is thus convenient to describe the atmosphere structure in a coordinate system $\{q^i|_{i=1}^3\}$, where $q^1(x) = \phi(x)$. This is the idea of Roche coordinates introduced by Kopal (1970). It would be naturally pleasant to find a 3-orthogonal system of this kind. However, the requirement that such coordinates exist (Cayley's (1872) problem), imposes a condition (the Cayley–Darboux equation, Darboux, 1898) which must be satisfied by function ϕ. The validity of this condition for Roche's binary potential was investigated by Kopal and Ali (1971) with a negative result. Unfortunately,

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this important result of non-existence of any 3-orthogonal Roche coordinates was not appropriately pointed out in recent reviews (e.g., Kopal, 1972) and the publicity of the computations by Kitamura (1970), which are actually performed only in the planes of symmetry, caused confusion. This misunderstanding has led the present author to deriving the radiative transfer equation in general orthogonal coordinates (Hadrava, 1981), which is actually applicable only to single rotating stars.

In Section 2 the Cayley problem is reinvestigated and the Roche coordinates η, ξ are defined as continuations of asymptotically (at the centre of star) spherical coordinates, which are perpendicular to the equipotential layers, but not mutually perpendicular. In Section 3 the explicit form of these coordinates and the corresponding metric are given for binaries with eccentric orbits and general rotation.

2. The Cayley Problem

Let us construct a coordinate system $\{q^i(x)|_{i=1}^3\} = \{\phi, \eta, \xi\}$, where coordinate lines η = const. and ξ = const. are perpendicular to the equipotential surfaces of a given potential $\phi(x)$. The transformation from Cartesian coordinates $\{x^i|_{i=1}^3\}$ to $\{q^i|_{i=1}^3\}$ is fully determined by the dependence of $\eta(x)$ and $\xi(x)$ on a particular surface $\phi = \phi_0$ = const. and by the equations of coordinate lines

$$\left[\frac{\mathrm{d}x^i}{\mathrm{d}\phi}\right]_{\eta,\, \xi = \mathrm{const.}} = \frac{\nabla^i \phi}{(\nabla\phi)^2} . \tag{2.1}$$

It is always possible to choose η and ξ mutually perpendicular

$$0 = g_{23} = \frac{\partial x^i}{\partial \eta} \frac{\partial x^i}{\partial \xi} \tag{2.2}$$

on surface $\phi = \phi_0$. This can be achieved, e.g., by an arbitrary choice of η, and ξ is then determined by condition (2.2). However, to ensure the conservation of the perpendicularity of η and ξ on the other equipotential surfaces, condition (2.2) must be invariant with respect to motion (2.1), i.e., the condition

$$0 = \left[\frac{\mathrm{d}}{\mathrm{d}\phi} g_{23}\right]_{\eta,\, \xi = \mathrm{const.}} = \frac{\partial x^i}{\partial \eta}\left[\nabla^j\left(\frac{\nabla^i\phi}{(\nabla\phi)^2}\right) + \nabla^j\left(\frac{\nabla^i\phi}{(\nabla\phi)^2}\right)\right]\frac{\partial x^j}{\partial \xi} =$$
$$= \frac{2}{(\nabla\phi)^2}\frac{\partial x^i}{\partial \eta}\phi_{ij}\frac{\partial x^j}{\partial \xi} \tag{2.3}$$

must be satisfied. This condition, together with the conditions of orthogonality

$$\phi_i \frac{\partial x^i}{\partial \eta} = 0 , \tag{2.4a}$$

$$\phi_i \frac{\partial x^i}{\partial \xi} = 0 , \tag{2.4b}$$

select the η- and ξ-coordinate lines in the directions of the eigenvectors of projection of operator ϕ_{ij} onto the tangent plane to the equipotential. To ensure the invariance of these directions with respect to motion (1),

$$0 = \left[\frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{\partial x^i}{\partial\eta}\,\phi_{ij}\,\frac{\partial x^j}{\partial\xi}\right)\right]_{\eta,\,\xi=\mathrm{const.}} =$$

$$= \frac{\partial x^i}{\partial\eta}\left[\nabla^i\left(\frac{\phi_k}{(\nabla\phi)^2}\right)\phi_{kj} + \phi_{ijk}\,\frac{\phi_k}{(\nabla\phi)^2} + \phi_{ik}\nabla^j\left(\frac{\phi_k}{(\nabla\phi)^2}\right)\right]\frac{\partial x^j}{\partial\xi} =$$

$$= \frac{1}{(\nabla\phi)^4}\,\frac{\partial x^i}{\partial\eta}\,A_{ij}\,\frac{\partial x^j}{\partial\xi}\,, \tag{2.5}$$

where

$$A_{ij} = (\phi_{ijk}\phi_k + 2\phi_{ik}\phi_{kj})\phi_l\phi_l - 4\phi_{ik}\phi_k\phi_{jl}\phi_l\,, \tag{2.5a}$$

must be satisfied. Operators A_{ij} and ϕ_{ij} must thus have common eigenvectors $(\partial x^i/\partial q^j)|_{j=2}^{3}$ in the tangent plane to the equipotential surface. This is possible only if they commutate, i.e., if

$$0 = B = P(AP\phi - \phi PA)P\,, \tag{2.6}$$

where

$$P_{ij} = \delta_{ij} - \frac{\phi_i\phi_j}{(\nabla\phi)^2} \tag{2.7}$$

is the projector to the tangent plane to ϕ = const. There is only one linearly independent component of Equation (2.6)

$$0 = \varepsilon^{ijk}\phi_i B_{jk} + 2\varepsilon^{ijk}\phi_i A_{jl}P_{lm}\phi_{mk} =$$

$$= 2\varepsilon^{ijk}\phi_i[\phi_{jlm}\phi_m(\phi_{lk}\phi_n\phi_n - \phi_l\phi_{kn}\phi_n) - 2\phi_{jm}\phi_m\phi_{kl}\phi_{ln}\phi_n]\,. \tag{2.8}$$

It can be proved by straightforward calculation of all 222 terms of this summation (138 of them contain the third derivatives of ϕ), that this equation is identical with the well-known Cayley–Darboux equation, which has the form of a determinant of a 6 × 6 matrix. This equation can be obtained as the condition of existence of a nontrivial solution of homogeneous linear equations in $(\partial x^{(i}/\partial\eta)(\partial x^{j)}/\partial\xi)$, which follows from Equations (2.2) and (2.4) and their derivatives (see, e.g., Kagan, 1948).

Another approach was chosen by Kopal and Ali (1971). Their procedure can be briefly summarized, if

$$\frac{\partial x^i}{\partial\eta} \sim \varepsilon^{ijk}\phi_j\,\frac{\partial x^k}{\partial\xi} \tag{2.9}$$

is expressed from Equations (2.2) and (2.4a). Substituting this into Equation (2.3), one obtains a quadratic equation for $\partial x/\partial \xi$, which (together with Equation (2.4b)) forms a homogeneous set. Its solution is of the form

$$\frac{\partial x^i}{\partial \xi} \sim X^i = \varepsilon^{ij3} D^{1/2} \phi_j + 2\varepsilon^{ijk} \varepsilon^{lm3} \phi_j \phi_l a_{km} , \tag{2.10}$$

where

$$a_{ij} = a_{ji} = \varepsilon_{(ikl} \phi_k \phi_{lj)} \tag{2.10a}$$

and

$$D = \sum_{ijk} |\varepsilon_{ijk}| \, [2(a_{jk}^2 - a_{jj} a_{kk}) \phi_i^2 - 4(a_{ij} a_{ik} - a_{ii} a_{jk}) \phi_j \phi_k] . \tag{2.10b}$$

The condition of integrability of Equation (2.10) as a partial differential equation for $\xi = \xi(x)$ is then

$$0 = \varepsilon_{ijk} X_i X_{j,k} . \tag{2.11}$$

Unfortunately, owing to the complicated form of these expressions, any attempt to prove the equivalence of this method with the two mentioned above fail.

It should be pointed out once more, that the incompatibility of a given potential with the Cayley–Darboux equation (2.8) does not mean only a technical problem in finding an analytical expression for the Roche coordinates (as defined by Kopal, 1970), but it means the principial impossibility of the existence of any orthogonal coordinates $\{q^i\} = \{\phi, \eta, \xi\}$. Since the Roche potential can only be stationary in corotating, i.e., non-inertial coordinates $\{x, y, z\}$, the Roche coordinates cannot be generally time-orthogonal either. It is thus not worth insisting on condition (2.2), but it is possible to define the Roche coordinates by Equation (2.1), which ensures the validity of condition (2.4), and to choose initial conditions on a surface $\phi = \phi_0$ (or $\phi_0 \to -\infty$), e.g., in the form of spherical coordinates.

3. Roche Coordinates in Binaries

Let ϕ be the Roche potential of a binary component (see Wilson, 1979),

$$\phi = -\frac{\Omega^2 R^2}{(1+Q)} \left[\frac{R}{r} + \frac{QR}{|r-D|} - \frac{QR}{D} - \frac{QR}{D^2} r_D^2 + \frac{(1+Q)P^2}{2R^2} (r^2 - r_\omega^2) \right] = \sum_{k=-1}^{\infty} a_k(\mathbf{n}) r^k =$$

$$= -\frac{\Omega^2 R^2}{(1+Q)} \left[\frac{R}{r} + \frac{QR}{2D^3} (3r_D^2 - r^2) + \frac{(1+Q)P^2}{2R^2} (r^2 - r_\omega^2) + \frac{QR}{D} \sum_{k=3}^{\infty} \left(\frac{r}{D}\right)^k P_k(n_D) \right], \tag{3.1}$$

where Ω is the mean orbital angular velocity, R is the semi-major axis of the binary orbit, $Q = M_2/M_1$ the mass ratio, D the instantaneous separation of the secondary star, $\mathbf{i}_D = \mathbf{D}/D$ the unit vector in its direction, $r_D = (\mathbf{i}_D \mathbf{r})$ the projection of radius vector $\mathbf{r}$ onto

$\mathbf{i}_D$, $P = \omega/\Omega$, where ω is the rotational angular velocity of the primary star (sidereal – i.e., with respect to an inertial system), $\mathbf{i}_\omega = \boldsymbol{\omega}/\omega$, and $\mathbf{r}_\omega = (\mathbf{i}_\omega \mathbf{r})$ the projection of $\mathbf{r}$ onto $\mathbf{i}_\omega$ (which is not necessarily perpendicular to the orbital plane), $\mathbf{n} = \mathbf{r}/r$. The third term (QR/D) in brackets of the first form of Equation (3.1) is the potential in the centre of the primary due to the secondary star. It can be subtracted, because it is constant at a given moment. The zero value of the physically ambiguous coefficient a_0 will simplify later calculations.

Naturally, only if $R = D$, $P = 1$, $\boldsymbol{\omega} \parallel \boldsymbol{\Omega}$ (zero eccentricity, synchronous rotation perpendicular to the orbital plane) or $Q = 0$ (rotating single star), can hydrostatic equilibrium of the primary be achieved exactly. However, form (3.1) of the potential includes both these cases and, in a general case of simultaneous rotation and tides (like in the Earth–Moon system), it can yield a better approximation to the shape of the primary than neglecting one of these effects.

If potential (3.1) is substituted into Equation (2.8), one find that the Cayley–Darboux equation is satisfied only if $\boldsymbol{\omega} \parallel \mathbf{D}$. The special cases are $Q = 0$ (rotating single star) and/or $P = 0$ (i.e., zero sidereal rotation). Cayley's problem is trivial for $\boldsymbol{\omega} \parallel \mathbf{D}$, because rotational symmetry with respect to this axis decreases its dimension. In a general case Equation (2.8) is valid only on special surfaces. Due to symmetry, one of them is the plane spanned by $\mathbf{i}_D$ and $\mathbf{i}_\omega$ and, if they are mutually perpendicular, the equatorial plane is also. Kitamura (1970) carried out the computations for these very planes.

The gradient of potential (3.1) is

$$\nabla\phi = \sum_{k=-1}^{\infty} \mathbf{A}_k(\mathbf{n}) r^{k-1} = -\frac{\Omega^2 R^2}{(1+Q)} \left[-\frac{R}{r^3}\mathbf{r} + \frac{QR}{D^3}(3\mathbf{i}_D r_D - \mathbf{r}) + \frac{(1+Q)P^2}{R^2}(\mathbf{r} - \mathbf{i}_\omega r) + \right.$$

$$\left. + \frac{QR}{D^2} \sum_{k=3}^{\infty} \left(\frac{r}{D}\right)^{k-1} (\mathbf{i}_D P_k' - \mathbf{n} P_{k-1}) \right]. \tag{3.2}$$

The right-hand side of Equation (2.1) for coordinate lines η, ξ = const. can also be expanded into power series in $\mathbf{r}$. This equation then reads

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\phi} = \sum_{k=2}^{\infty} \mathbf{D}_k(\mathbf{n}) r^k, \tag{3.3}$$

where coefficients $\mathbf{D}_k$ are given by the recurrent formula

$$\mathbf{D}_k = \left(\mathbf{A}_{k-3} - \sum_{j=-3}^{k-3} \mathbf{D}_{k-j-4} \sum_{l=-1}^{j+3} (\mathbf{A}_l \mathbf{A}_{j-l+2}) \right) \Big/ A_{-1}^2 \tag{3.4}$$

(specifically $\mathbf{D}_2 = \mathbf{A}_{-1}/A_{-1}^2$). Let us look for the solution of Equation (3.3) in the form

$$\mathbf{r} = \sum_{k=1}^{\infty} \mathbf{B}_k(\eta, \xi) \phi^{-k}. \tag{3.5}$$

Substituting this assumption into Equation (3.3) and comparing the terms of equal order in ϕ, we can find a recurrent set of equations for coefficients $\mathbf{B}_k$. The first one

$$-\mathbf{B}_1 = -\frac{1+Q}{\Omega^2 R^3}\,\mathbf{B}_1\,|B_1| \tag{3.6a}$$

has the solution

$$\mathbf{B}_1 = \frac{\Omega^2 R^3}{1+Q}\,\mathbf{e}\,, \tag{3.6}$$

where $\mathbf{e} = \mathbf{e}(\eta, \xi)$ is a unit vector in the initial (i.e., for $r \to 0$) direction of the coordinate line ($\mathbf{e}$ is arbitrarily parametrized by η and ξ). The equation for the second coefficient

$$-2\mathbf{B}_2 = -\mathbf{B}_2 \cdot \mathbf{e}(\mathbf{e} \cdot \mathbf{B}_2) \tag{3.7a}$$

is a singular homogeneous equation. Its general solution

$$\mathbf{B}_2 = C\mathbf{e}\,, \tag{3.7b}$$

contains an integration constant C, which must be determined from the condition that $\phi = \phi(\mathbf{r})$ is one component of the inverse transformation to $\mathbf{r} = \mathbf{r}(\phi, \eta, \xi)$. The assumption $a_0 = 0$ chosen above implies $C = 0$, i.e.,

$$\mathbf{B}_2 = 0\,. \tag{3.7}$$

All equations for subsequent coefficients $\mathbf{B}_k$ are also linear but non-singular. The next one is still homogeneous (since $\mathbf{A}_1 = 0$, $\mathbf{B}_2 = 0$)

$$(2.1 - \mathbf{e} \otimes \mathbf{e})\mathbf{B}_3 = 0\,, \tag{3.8a}$$

hence, its solution is

$$\mathbf{B}_3 = 0\,. \tag{3.8}$$

The equation

$$(3.1 - \mathbf{e} \otimes \mathbf{e})\mathbf{B}_4 = \frac{\Omega^8 R^{11}}{(1+Q)^4}\left[\frac{QR}{D^3}\,(3e_D\mathbf{i}_D + (1 - 6e_D^2)\mathbf{e} + \frac{(1+Q)P^2}{R^2}\,((2e_\omega^2 - 1)\mathbf{e} - e_\omega \mathbf{i}_\omega)\right], \tag{3.9a}$$

where $\mathbf{e}_{D,\,\omega} = (\mathbf{i}_{D,\,\omega}\mathbf{e})$, admits of the solution.

$$\mathbf{B}_4 = \frac{\Omega^8 R^{11}}{6(1+Q)^4}\left[3\,\frac{QR}{D^3}\,(2e_D\mathbf{i}_D - (5e_D^2 - 1)\mathbf{e}) + \frac{(1+Q)P^2}{R^2}\,((5e_\omega^2 - 3)\mathbf{e} - 2e_\omega\mathbf{i}_\omega)\right]. \tag{3.9b}$$

In a similar way

$$\mathbf{B}_5 = \frac{Q\Omega^{10}R^{15}}{8(1+Q)^5 D^4}\,[3(5e_D^2 - 1)\mathbf{i}_D - 5e_D(7e_D^2 - 3)\mathbf{e}]\,, \tag{3.10}$$

$$\mathbf{B}_6 = \frac{Q\Omega^{12}R^{18}}{8(1+Q)^6 D^5}\,[4e_D(7e_D^2 - 3)\mathbf{i}_D - 3(21e_D^4 - 14e_D^2 + 1)\mathbf{e}]\,, \tag{3.11}$$

etc. The analytic expression for the Roche coordinates can thus be found to an arbitrary order in $\phi^{-1}\,(\sim r)$.

The covariant components of the metric tensor in the coordinate base $\{\partial_{qi}\} = \{\partial_\phi, \partial_\eta, \partial_\xi\}$ can be found from transformation (3.5)

$$g_{ij} = \frac{\partial r^k}{\partial q^i}\,\delta_{kl}\,\frac{\partial r^l}{\partial q^j}\,. \tag{3.12}$$

By substituting Equations (3.5) to (3.11) into (3.12), we find that

$$\begin{aligned} g_{11} &= \sum_{n=1}^{\infty} \phi^{-n-3} \sum_{k=1}^{n} k(n-k+1)\,(\mathbf{B}_k\cdot\mathbf{B}_{n-k+1}) = \\ &= \phi^{-4}\,\frac{\Omega^4 R^6}{(1+Q)^2} + \phi^{-7}\,4\,\frac{\Omega^{10}R^{14}}{(1+Q)^5}\left[\frac{QR}{D^3}\,(1-3e_D^2) + \frac{(1+Q)P^2}{R^2}\,(e_\omega^2 - 1)\right] + \\ &\quad + \phi^{-8}\,\frac{Q\Omega^{12}R^{19}}{2(1+Q)^6 D^4}\,e_D(3 - 5e_D^2) + \\ &\quad + \phi^{-9}\,\frac{Q\Omega^{14}R^{22}}{8(1+Q)^7 D^5}\,(-35e_D^4 + 30e_D^2 - 3) + \sigma(\phi^{-10})\,. \end{aligned} \tag{13}$$

It can be proved that, consistently with the construction of solution (3.5) described above,

$$g_{1i} = -\sum_{n=1}^{\infty} \phi^{-n-2} \sum_{k=1}^{n} k\left(\mathbf{B}_k\,\frac{\partial\mathbf{B}_{n-k+1}}{\partial q^i}\right) = 0 \quad \text{for} \quad i = 2, 3\,. \tag{3.14}$$

Finally,

$$\begin{aligned} g_{ij} &= \sum_{n=1}^{\infty} \phi^{-n-1} \sum_{k=1}^{n} \left(\frac{\partial\mathbf{B}_k}{\partial q_i}\,\frac{\partial\mathbf{B}_{n-k+1}}{\partial q_j}\right) = \\ &= \phi^{-2}\,\frac{\Omega^4R^6}{(1+Q)^2}\,\gamma_{ij} + \phi^{-5}\,\frac{\Omega^{10}R^{14}}{3(1+Q)^5}\left[3\,\frac{QR}{D^3}\,(2e_{Di}e_{Dj} - (1+5e_D^2)\gamma_{ij}) + \right. \\ &\quad \left. + \frac{(1+Q)P^2}{R^2}\,((5e_\omega^2 - 3)\gamma_{ij} - 2e_{\omega i}e_{\omega j})\right] + \\ &\quad + \phi^{-6}\,\frac{5Q\Omega^{10}R^{18}}{4(1+Q)^6D^4}\,[6e_De_{Di}e_{Dj} - e_D(7e^{2l_D} - 3)\gamma_{ij}] + \end{aligned}$$

$$+ \phi^{-7} \frac{3Q\Omega^{14}R^{21}}{4(1+Q)^7 D^5} [4(7e_D^2 - 1)e_{Di}e_{Dj} - (21e_D^4 - 14e_D^2 + 1)\gamma_{ij}] + \sigma(\phi^{-8})$$

for $i, j = 2, 3$, (3.15)

where

$$\gamma_{ij} = \left(\frac{\partial \mathbf{e}}{\partial q^i} \frac{\partial \mathbf{e}}{\partial q^j}\right), \qquad e_{Di} = \left(\mathbf{i}_D \frac{\partial \mathbf{e}}{\partial q^i}\right), \qquad e_{\omega i} = \left(\mathbf{i}_\omega \frac{\partial \mathbf{e}}{\partial q^i}\right); \tag{3.15a}$$

are coefficients dependent on the way of parametrisation $\mathbf{e} = \mathbf{e}(\eta, \xi)$. If η and ξ are chosen as spherical coordinates, then

$$\gamma_{ij} = \mathrm{diag}(1, \cos^2 \eta) . \tag{3.16a}$$

If, moreover, the pole ($\eta = \pi/2$) coincides with $\mathbf{i}_D$, then

$$e_D = \sin \eta , \qquad e_{D1} = \cos \eta , \qquad e_{D2} = 0 , \tag{3.16b}$$

and if the zero longitude is chosen in the plane $(\mathbf{i}_D, \mathbf{i}_\omega)$, then

$$e_\omega = \cos \alpha \sin \eta + \sin \alpha \cos \eta \cos \xi , \qquad e_{\omega 1} = \cos \alpha \cos \eta - \sin \alpha \sin \eta \cos \xi ,$$

$$e_{\omega 2} = - \sin \alpha \cos \eta \sin \xi , \tag{3.16c}$$

where α is the angle between $\mathbf{i}_D$ and $\mathbf{i}_\omega$. The nondiagonal component of the metric is, in this case,

$$g_{12} = - \phi^{-5} \frac{2\Omega^{10}R^{12}P^2}{3(1+Q)^4} e_{\omega 1} e_{\omega 2} + \sigma(\phi^{-8}) . \tag{3.16}$$

The Roche coordinates are thus orthogonal in plane $(\mathbf{i}_D, \mathbf{i}_\omega)$, where $\sin \xi = 0$ and, hence, $e_{\omega 2} = 0$, and on the surface (where $e_{\omega 1} = 0$) of a two-sided funnel, to which $\mathbf{i}_D$ and $\mathbf{i}_\omega$ are tangent vectors (this surface approaches planes $(\mathbf{i}_D, \mathbf{i}_D \wedge \mathbf{i}_\omega)$ and $(\mathbf{i}_\omega, \mathbf{i}_D \wedge \mathbf{i}_\omega)$ if $\alpha = \pi/2$).

4. Conclusions

The Roche coordinates can be defined by Equation (2.1). This definition abandons the orthogonality of coordinates η and ξ, but it ensures their existence. The transformation from Roche to Cartesian coordinates can be found in the form of power series (3.5). The first six coefficients $\mathbf{B}_k$ are explicitly given by Equations (3.6) and (3.11), and the metric is given to the same degree of accuracy by Equations (3.13) and (3.15).

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